1. Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games,

(3)

(b) fewer than half of the games.

(2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses.

(2)

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games.

(3)
(Total 10 marks)

2. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)
(Total 15 marks)

3. A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.

(a) Write down a suitable model for the distribution of the number of faulty DVD players in a batch.

(2)

Find the probability that a batch contains

(b) no faulty DVD players,

(2)

(c) more than 4 faulty DVD players.

(2)

(d) Find the mean and variance of the number of faulty DVD players in a batch.

(2)
(Total 8 marks)

4. (a) Define the critical region of a test statistic.

(2)

A discrete random variable x has a Binomial distribution B(30, p). A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)
(Total 10 marks)

5. A bag contains a large number of counters of which 15% are coloured red. A random sample of 30 counters is selected and the number of red counters is recorded.

(a) Find the probability of no more than 6 red counters in this sample.

(2)

A second random sample of 30 counters is selected and the number of red counters is recorded.

(b) Using a Poisson approximation, estimate the probability that the total number of red counters in the combined sample of size 60 is less than 13.

(3)
(Total 5 marks)

6. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.

(2)

(b) Find the probability that there are at least 2 defective components in the box.

(3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.

(4)
(Total 9 marks)

7. In a large college 58% of students are female and 42% are male. A random sample of 100 students is chosen from the college. Using a suitable approximation find the probability that more than half the sample are female.

(Total 7 marks)

8. Each cell of a certain animal contains 11000 genes. It is known that each gene has a probability 0.0005 of being damaged.

A cell is chosen at random.

(a) Suggest a suitable model for the distribution of the number of damaged genes in the cell.

(2)

(b) Find the mean and variance of the number of damaged genes in the cell.

(2)

(c) Using a suitable approximation, find the probability that there are at most 2 damaged genes in the cell.

(4)
(Total 8 marks)

9. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head.

(2)

Find the probability that Sue records

(b) exactly 8 heads,

(2)

(c) at least 4 heads.

(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.

(6)
(Total 12 marks)

10. The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

(a) exactly 2 faulty bolts,

(2)

(b) more than 3 faulty bolts.

(2)

These bolts are sold in bags of 20. John buys 10 bags.

(c) Find the probability that exactly 6 of these bags contain more than 3 faulty bolts.

(3)
(Total 7 marks)

11. (a) Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution.

(2)

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01

(b) Find the probability that 2 consecutive calls will be connected to the wrong agent.

(2)

(c) Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.

(3)

The call centre receives 1000 calls each day.

(d) Find the mean and variance of the number of wrongly connected calls.

(3)

(e) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.

(2)
(Total 12 marks)

12. (a) (i) Write down two conditions for $X \sim \text{Bin}(n, p)$ to be approximated by a normal distribution $Y \sim \text{N}(\mu, \sigma^2)$.

(2)

(ii) Write down the mean and variance of this normal approximation in terms of n and p.

(2)

A factory manufactures 2000 DVDs every day. It is known that 3% of DVDs are faulty.

(b) Using a normal approximation, estimate the probability that at least 40 faulty DVDs are produced in one day.

(5)

The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs £0.70 to manufacture a DVD and the factory sells non-faulty DVDs for £11.

(c) Find the expected profit made by the factory per day.

(3)
(Total 12 marks)

13. The random variable J has a Poisson distribution with mean 4.

(a) Find $P(J \geq 10)$.

(2)

The random variable K has a binomial distribution with parameters $n = 25$, $p = 0.27$.

(b) Find $P(K \leq 1)$.

(3)
(Total 5 marks)

14. For a particular type of plant 45% have white flowers and the remainder have coloured flowers. Gardenmania sells plants in batches of 12. A batch is selected at random.

Calculate the probability that this batch contains

(a) exactly 5 plants with white flowers,

(3)

(b) more plants with white flowers than coloured ones.

(2)

Gardenmania takes a random sample of 10 batches of plants.

(c) Find the probability that exactly 3 of these batches contain more plants with white flowers than coloured ones.

(3)

Due to an increasing demand for these plants by large companies, Gardenmania decides to sell them in batches of 50.

(d) Use a suitable approximation to calculate the probability that a batch of 50 plants contains more than 25 plants with white flowers.

(7)
(Total 15 marks)

15. The continuous random variable L represents the error, in mm, made when a machine cuts rods to a target length. The distribution of L is continuous uniform over the interval [–4.0, 4.0].

Find

(a) P(L< – 2.6),

(1)

(b) P(L < – 3.0 or L > 3.0).

(2)

A random sample of 20 rods cut by the machine was checked.

(c) Find the probability that more than half of them were within 3.0 mm of the target length.

(4)
(Total 7 marks)

16. A manufacturer produces large quantities of coloured mugs. It is known from previous records that 6% of the production will be green.

A random sample of 10 mugs was taken from the production line.

(a) Define a suitable distribution to model the number of green mugs in this sample.

(1)

(b) Find the probability that there were exactly 3 green mugs in the sample.

(3)

A random sample of 125 mugs was taken.

(c) Find the probability that there were between 10 and 13 (inclusive) green mugs in this sample, using

(i) a Poisson approximation,

(3)

(ii) a Normal approximation.

(6)
(Total 13 marks)

17. A fair coin is tossed 4 times.

Find the probability that

(a) an equal number of heads and tails occur,

(2)

(b) all the outcomes are the same,

(3)

(c) the first tail occurs on the third throw.

(2)
(Total 7 marks)

18. The random variable $X \sim \mathrm{B}(150, 0.02)$.

Use a suitable approximation to estimate $\mathrm{P}(X > 7)$.

(Total 4 marks)

19. It is estimated that 4% of people have green eyes. In a random sample of size n, the expected number of people with green eyes is 5.

(a) Calculate the value of n.

(3)

The expected number of people with green eyes in a second random sample is 3.

(b) Find the standard deviation of the number of people with green eyes in this second sample.

(4)
(Total 7 marks)

20. A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor's records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

(2)

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients.

(2)

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

(c) Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.
(6)

(d) From a sample of size 20, find the greatest number of patients who need to recover from the test in part (c) to be significant at the 1% level.
(4)
(Total 14 marks)

21. The random variables R, S and T are distributed as follows

$$R \sim \mathrm{B}(15, 0.3), \quad S \sim \mathrm{Po}(7.5), \quad T \sim \mathrm{N}(8, 2^2).$$

Find

(a) $\mathrm{P}(R = 5)$,
(2)

(b) $\mathrm{P}(S = 5)$,
(1)

(c) $\mathrm{P}(T = 5)$.
(1)
(Total 4 marks)

22. From company records, a manager knows that the probability that a defective article is produced by a particular production line is 0.032.

A random sample of 10 articles is selected from the production line.

(a) Find the probability that exactly 2 of them are defective.
(3)

On another occasion, a random sample of 100 articles is taken.

(b) Using a suitable approximation, find the probability that fewer than 4 of them are defective.

(4)

At a later date, a random sample of 1000 is taken.

(c) Using a suitable approximation, find the probability that more than 42 are defective.

(6)
(Total 13 marks)

23. (a) State two conditions under which a random variable can be modelled by a binomial distribution.

(2)

In the production of a certain electronic component it is found that 10% are defective.

The component is produced in batches of 20.

(b) Write down a suitable model for the distribution of defective components in a batch.

(1)

Find the probability that a batch contains

(c) no defective components,

(2)

(d) more than 6 defective components.

(2)

(e) Find the mean and the variance of the defective components in a batch.

(2)

A supplier buys 100 components. The supplier will receive a refund if there are more than 15 defective components.

(f) Using a suitable approximation, find the probability that the supplier will receive a refund.

(4)
(Total 13 marks)

24. The random variable R has the binomial distribution B(12, 0.35).

(a) Find $P(R \geq 4)$.

(2)

The random variable S has the Poisson distribution with mean 2.71.

(b) Find $P(S \leq 1)$.

(3)

The random variable T has the normal distribution $N(25, 5^2)$.

(c) Find $P(T \leq 18)$.

(2)
(Total 7 marks)

25. The discrete random variable X is distributed $B(n, p)$.

(a) Write down the value of p that will give the most accurate estimate when approximating the binomial distribution by a normal distribution.

(1)

(b) Give a reason to support your value.

(1)

(c) Given that $n = 200$ and $p = 0.48$, find $P(90 \leq X < 105)$.

(7)
(Total 9 marks)

26. (a) Write down two conditions needed to be able to approximate the binomial distribution by the Poisson distribution.

(2)

A researcher has suggested that 1 in 150 people is likely to catch a particular virus.

Assuming that a person catching the virus is independent of any other person catching it,

(b) find the probability that in a random sample of 12 people, exactly 2 of them catch the virus.

(4)

(c) Estimate the probability that in a random sample of 1200 people fewer than 7 catch the virus.

(4)
(Total 10 marks)

27. In a town, 30% of residents listen to the local radio station. Four residents are chosen at random.

(a) State the distribution of the random variable X, the number of these four residents that listen to local radio.

(2)

(b) On graph paper, draw the probability distribution of X.

(3)

(c) Write down the most likely number of these four residents that listen to the local radio station.

(1)

(d) Find E(X) and Var (X).

(3)
(Total 9 marks)

28. (a) Write down the conditions under which the binomial distribution may be a suitable model to use in statistical work.

(4)

A six-sided die is biased. When the die is thrown the number 5 is twice as likely to appear as any other number. All the other faces are equally likely to appear. The die is thrown repeatedly.

Find the probability that

(b) (i) the first 5 will occur on the sixth throw,

(ii) in the first eight throws there will be exactly three 5s.

(8)
(Total 12 marks)

29. A farmer noticed that some of the eggs laid by his hens had double yolks. He estimated the probability of this happening to be 0.05. Eggs are packed in boxes of 12.

Find the probability that in a box, the number of eggs with double yolks will be

(a) exactly one,

(3)

(b) more than three.

(2)

A customer bought three boxes.

(c) Find the probability that only 2 of the boxes contained exactly 1 egg with a double yolk.

(3)

The farmer delivered 10 boxes to a local shop.

(d) Using a suitable approximation, find the probability that the delivery contained at least 9 eggs with double yolks.

(4)

The weight of an individual egg can be modelled by a normal distribution with mean 65 g and standard deviation 2.4 g.

(e) Find the probability that a randomly chosen egg weighs more than 68 g.

(3)
(Total 15 marks)

30. The continuous random variable X represents the error, in mm, made when a machine cuts piping to a target length. The distribution of X is rectangular over the interval [–5.0, 5.0].

Find

(a) $P(X < -4.2)$,

(1)

(b) $P(|X| < 1.5)$.

(2)

A supervisor checks a random sample of 10 lengths of piping cut by the machine.

(c) Find the probability that more than half of them are within 1.5 cm of the target length.

(3)

If $X < -4.2$, the length of piping cannot be used. At the end of each day the supervisor checks a random sample of 60 lengths of piping.

(d) Use a suitable approximation to estimate the probability that no more than 2 of these lengths of piping cannot be used.

(5)
(Total 11 marks)

31. On a typical weekday morning customers arrive at a village post office independently and at a rate of 3 per 10 minute period.

Find the probability that

(a) at least 4 customers arrive in the next 10 minutes,

(2)

(b) no more than 7 customers arrive between 11.00 a.m. and 11.30 a.m.

(3)

The period from 11.00 a.m. to 11.30 a.m. next Tuesday morning will be divided into 6 periods of 5 minutes each.

(c) Find the probability that no customers arrive in at most one of these periods.

(6)

The post office is open for $3\frac{1}{2}$ hours on Wednesday mornings.

(d) Using a suitable approximation, estimate the probability that more than 49 customers arrive at the post office next Wednesday morning.

(7)

(Total 18 marks)

1. (a) Let X be the random variable the number of games Bhim loses.

$X \sim B(9, 0.2)$ B1

$P(X \le 3) - P(X \le 2) = 0.9144 - 0.7382$ or $(0.2)^3(0.8)^6\dfrac{9!}{3!6!}$ M1

$= 0.1762$ $= 0.1762$

awrt 0.176 A1 3

Note

B1 – writing or use of B(9, 0.2)

M1 for writing/ using $P(X \le 3) - P(X \le 2)$ or $(p)^3(1-p)^6\dfrac{9!}{3!6!}$

A1 awrt 0.176

Special case : Use of Po(1.8)

can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\dfrac{e^{-1.8}1.8^3}{3!}$

or awrt to 0.161

If B(9, 0.2) is not seen then the only mark available for using Poisson is M1.
(b) can get M1 A0 – M1 for writing or using $P(X \le 4)$ or may be implied by awrt 0.964

(b) $P(X \le 4) = 0.9804$ awrt 0.98 M1 A1 2

Note

M1 for writing or using $P(X \le 4)$
A1 awrt 0.98

Special case : Use of Po(1.8)

can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\dfrac{e^{-1.8}1.8^3}{3!}$

or awrt to 0.161

If B(9, 0.2) is not seen then the only mark available for using Poisson is M1.
(b) can get M1 A0 – M1 for writing or using $P(X \le 4)$ or may be implied by awrt 0.964

(c) Mean = 3 variance = 2.85, $\dfrac{57}{20}$ B1 B1 2

Note

B1 3
B1 2.85, or exact equivalent

(d) Po(3) poisson M1

$P(X > 4) = 1 - P(X \le 4)$ M1

$= 1 - 0.8153$

		$= 0.1847$	A1	3

Note

M1 for using Poisson
M1 for writing or using $1 – P(X \le 4)$ NB $P(X \le 4)$ is 0.7254 Po(3.5) and 0.8912 Po(2.5)
A1 awrt 0.185

Use of Normal

Can get M0 M1 A0 – for M1 they must write $1 – P(X \le 4)$ or get awrt 0.187

[10]

2.	(a)	2 outcomes/faulty or not faulty/success or fail	B1	
		A constant probability	B1	2
		Independence		
		Fixed number of trials (fixed n)		

Note

B1 B1 one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed.

	(b)	$X \sim B(50, 0.25)$	M1	
		$P(X \le 6) = 0.0194$		
		$P(X \le 7) = 0.0453$		
		$P(X \ge 18) = 0.0551$		
		$P(X \ge 19) = 0.0287$		
		CR $X \le 6$ and $X \ge 19$	A1 A1	3

Note

M1 for writing or using B(50, 0.25) also may be implied by both CR being correct. Condone use of P in critical region for the method mark.
A1 $(X) \le 6$ o.e. [0, 6] DO NOT accept $P(X \le 6)$
A1 $(X) \ge 19$ o.e. [19, 50] DO NOT accept $P(X \ge 19)$

	(c)	$0.0194 + 0.0287 = 0.0481$	M1 A1	2

Note

M1 Adding two probabilities for two tails. Both probabilities must be less than 0.5
A1 awrt 0.0481

	(d)	8(It) is not in the Critical region or 8(It) is not significant or $0.0916 > 0.025$;	M1	
		There is evidence that the probability of a faulty bolt is 0.25 or the company's claim is correct	A1ft	2

Note

M1 one of the given statements followed through from their CR.
A1 contextual comment followed through from their CR.
NB A correct contextual comment **alone** followed through from their CR.will get M1 A1

(e)	$H_0 : p = 0.25 \quad H_1 : p < 0.25$	B1 B1	
	$P(X \leq 5) = 0.0070$ or CR $X \leq 5$	M1 A1	
	$0.007 < 0.01$,		
	5 is in the critical region, reject H_0, significant.	M1	
	There is evidence that the probability of faulty bolts has decreased	A1ft	6

Note

B1 for H_0 must use p or π (pi)

B1 for H_1 must use p or π (pi)

M1 for finding or writing $P(X \leq 5)$ or attempting to find a critical region or a correct critical region

A1 awrt 0.007/CR $X \leq 5$

M1 correct statement using their Probability and 0.01 if one tail test or a correct statement using their Probability and 0.005 if two tail test.

The 0.01 or 0.005 needn't be explicitly seen but implied by correct statement compatible with their H_1. If no H_1 given M0

A1 correct contextual statement follow through from their prob and H1. Need faulty bolts and decreased.

NB A correct contextual statement **alone** followed through from their prob and H_1 get M1 A1

[15]

3.	(a)	$X \sim B(20, 0.05)$	B1 B1	2

Note

1st B1 for binomial

2nd B1 for 20 and 0.05 o.e

(b)	$P(X = 0) = 0.95^{20} = 0.3584859\ldots$ or 0.3585 using tables .	M1 A1	2

Note

M1 for finding $(p)^{20}$ $0 < p < 1$ this working needs to be seen if answer incorrect to gain the M1

A1 awrt 0.358 or 0.359.

(c)	$P(X > 4)$	$=1 - P\ 4(X \le 4)$	M1	
		$= 1 - 0.9974$		
		$= 0.0026$	A1	2

Note

M1 for writing $1 - P(X \le 4)$

or $1 - [\ P(X = 0) + P(X = 1)$
$+ P(X = 2) + P(X = 3) + P(X = 4)]$

or $1 - 0.9974$

or $1 - 0.9568$

A1 awrt 0.0026 or 2.6×10^{-3}, do not accept a fraction e.g. 26/10000

(d)	Mean = $20 \times 0.051 = 1$	B1	
	Variance = $20 \times 0.05.\times 0.95 = 0.95$	B1	2

Note

1st **B1** for 1

2nd **B1** for 0.95

NB In parts b, c and d correct answers with no working gain full marks

[8]

4.	(a)	The set of values of the test statistic for which	B1	
		the null hypothesis is rejected in a hypothesis test.	B1	2

Note

1st **B1** for **"values/ numbers"**

2nd **B1** for "reject the null hypothesis" o.e or the test is significant

(b)	$X \sim B(30,0.3)$	M1	
	$P(X \le 3) = 0.0093$		
	$P(X \le 2) = 0.0021$	A1	
	$P(X \ge 16) = 1 - 0.9936 = 0.0064$		
	$P(X \ge 17) = 1 - 0.9979 = 0.0021$	A1	
	Critical region is $(0 \le)x \le 2$ or $16 \le x(\le 30)$	A1A1	5

Note

M1 for using B(30,0.3)

1st A1 P($X \leq 2$) = 0.0021

2nd A1 0.0064

3rd A1 for (X) $\leq$ 2 or (X) < 3 They get **A0** if they write
P($X \leq 2$/ $X \leq 3$)

4th A1 (X) $\geq$ 16 or (X) > 15 They get **A0** if they write
P($X \geq 16$ $X \geq 15$

NB these are B1 B1 but mark as A1 A1

16$\leq$ $X \leq$ 2 etc is accepted

To describe the critical regions they can use any letter or no letter at all. It does not have to be X.

(c)	Actual significance level 0.0021+0.0064=0.0085 or 0.85%	B1	1

Note

B1 correct answer only

(d)	15 (it) is not in the critical region	Bft 2, 1, 0	
	not significant		
	No significant evidence of a change in P = 0.3 accept H_0, (reject H_1) P($x \geq 15$) = 0.0169		2

Note

Follow through 15 and their critical region

B1 for any one of the 5 correct statements up to a maximum of B2

– B1 for any incorrect statements

[10]

5.	(a)	[X ~ B(30, 0.15)]			
		P($X \leq 6$), = 0.8474	awrt 0.847	M1 A1	2

Note

M1 for a correct probability statement P($X \leq 6$) or P(X < 7) or P (X =0) + P (X = 1) + P(X=2)+ P (X =4) + P(X =5) + P(X =6). (may be implied by long calculation) Correct answer gets M1 A1. allow 84.74%

(b)	$Y \sim B(60, 0.15) \approx Po(9)$	for using Po(9)	B1	
	$P(Y < 12), = 0.8758$	awrt 0.876	M1 A1	3

[N.B. normal approximation gives 0.897, exact binomial gives 0.894]

Note

B1 may be implied by using Po(9). Common incorrect answer which implies this is 0.9261

M1 for a correct probability statement $P(X \leq 12)$ or $P(X < 13)$ or $P(X = 0) + P(X = 1) + \ldots + P(X = 12)$ (may be implied by long calculation) and attempt to evaluate this probability using their Poisson distribution.

Condone $P(X \leq 13) = 0.8758$ for B1 M1 A1

Correct answer gets B1 M1 A1

Use of normal or exact binomial get B0 M0 A0

[5]

6. (a) X represents the number of defective components.

$P(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1	2

Note

Normal distribution used. B1for mean only

(b)

$P(X \geq 2) = 1 - P(X \leq 1)$	M1	
$= 1 - (p)^{10} - \text{(a)}$	A1ft	
$= 0.0043$	A1	3

Note

Special case for parts a and b
If they use 0.1 do not treat as misread as it makes it easier.
(a) M1 A0 if they have 0.3874
(b) M1 A1ft A0 they will get 0.2639
(c) Could get B1 B0 M1 A0

(c) $X \sim \text{Po}(2.5)$ B1B

$P(1 \le X \le 4) = P(X \le 4) - P(X = 0)$ M1

$= 0.8912 - 0.0821$

$= 0.809$ A1 4

<u>Note</u>

For any other values of p which are in the table do not use misread.
Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c)
B1 B0 M1 A0

[9]

7. $X \sim B(100,0.58)$

$Y \sim N(58, 24.36)$ B1B1B1

$[P(X > 50) = P(X \ge 51)]$ using 50.5 or 51.5 or 49.5 or 48.5 M1

$= P\left(z \ge \pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their M1

μ and σ for M1

$= P(z \ge -1.52\ldots)$ A1

$= 0.9357$ A1 7

<u>alternative</u>

$X \sim B(100,0.58)$

$Y \sim N(42, 24.36)$ B1B1B1

$[P(X < 50) = P(X \le 49)]$ using 50.5 or 51.5 or 49.5 or 48.5 M1

$= P\left(z \le \pm\left(\frac{49}{5-42\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their M1A1

μ and σ for M1

$= P(z \le 1.52\ldots)$

$= 0.9357$ A1

The first 3 marks may be given if the following figures are seen in
the standardisation formula :– 58 or 42,
24.36 or √24.36 or √24.4 or awrt 4.94.
Otherwise
B1 normal
B1 58 or 42
B1 24.36
M1 using 50.5 or 51.5 or 49.5 or 48.5. ignore the direction of the inequality.
M1 standardising 50.5, 51, 51.5 ,48.5, 49, 49.5 and their μ and σ.
They may use √ 24 or √24.36 or √24.4 or awrt 4.94 for σ or the
√of their variance.

A1 ± 1.52. may be awarded for $\pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)$ or $\pm\left(\frac{49.5-42}{\sqrt{24.36}}\right)$ o.e.

A1 awrt 0.936

[7]

8. (a) $X \sim B(11000, 0.0005)$ M1 A1 2

M1 for Binomial,
A1 fully correct
These cannot be awarded unless seen in part a

(b) $E(X) = 11000 \times 0.0005 = 5.5$ B1
$\text{Var}(X) = 11000 \times 0.0005 \times (1 - 0.0005)$
$= 5.49725$ B1 2

B1 cao
B1 also allow 5.50, 5.497, 5.4973, do **not** allow 5.5

(c) $X \sim \text{Po}(5.5)$ M1A1
$P(X \leq 2) = 0.0884$ dM1 A1 4

M1 for Poisson
A1 for **using** Po (5.5)
M1 this is dependent on the previous M mark.
It is for attempting to find $P(X \leq 2)$
A1 awrt 0.0884
Correct answer with no working gets full marks

<u>Special case</u> If they use normal approximation they could get
M0 A0 M1 A0 if they use 2.5 in their standardisation.

NB exact binomial is 0.0883

[8]

9. (a) $X \sim B(15, 0.5)$ B1 B1 2

B1 for Binomial
B1 for 15 and 0.5 must be in part a
This need not be in the form written

(b) $P(X = 8) = P(X \leq 8) - P(X \leq 7)$ or $\left(\frac{15!}{8!7!}\right)(p)^8(1-p)^7$ M1
$= 0.6964 - 0.5$
$= 0.1964$ awrt 0.196 A1 2

M1 attempt to find $P(X = 8)$ any method. Any value of p
A1 awrt 0.196
Answer only full marks

(c) $P(X \geq 4) = 1 - P(X \leq 3)$ M1
$= 1 - 0.0176$
$= 0.9824$ A1 2

M1 for $1 - P(X \leq 3)$.
A1 awrt 0.982

(d) $H_0 : p = 0.5$ B1
$H_1 : p > 0.5$ B1

$X \sim B(15, 0.5)$

$P(X \geq 13) = 1 - P(X \leq 12)$	$[P(X \geq 12) = 1 - 0.9824 = 0.0176]$ att $P(X \geq 13)$	M1	
$= 1 - 0.9963$	$P(X \geq 13) = 1 - 0.9963 = 0.0037$		
$= 0.0037$	CR $X \geq 13$ awrt 0.0037/ CR $\times \geq 13$	A1	
$0.0037 < 0.01$	$13 \geq 13$		

Reject H_0 or it is significant or a correct statement in context from their values M1

There is sufficient evidence at the 1% significance level that the coin is biased in favour of heads A1 6
Or
There is evidence that Sues belief is correct

B1 for correct H_0. must use p or π
B1 for correct H_1 must be one tail must use p or π
M1 attempt to find $P(X \geq 13)$ correctly. E.g. $1 - P(X \leq 12)$
A1 correct probability or CR

To get the next 2 marks the null hypothesis must state or imply that $(p) = 0.5$

M1 for correct statement based on their probability or critical region or a correct contextualised statement that implies that. not just 13 is in the critical region.

A1 This depends on their M1 being awarded for rejecting H_0. Conclusion in context. Must use the words biased in favour of heads or biased against tails or sues belief is correct. NB this is a B mark on EPEN.

They may also attempt to find $P(X < 13) = 0.9963$ and compare with 0.99

[12]

10. (a) Let X be the random variable the number of faulty bolts M1

$P(X \leq 2) - P(X \leq 1) = 0.0355 - 0.0076$ or $(0.3)^2(0.7)^{18}\dfrac{20!}{18!2!}$ A1 2

$= 0.0279$ $= 0.0278$

M1 Either
attempting to use $P(X \leq 2) - P(X \leq 1)$
or attempt to use binomial and find $p(X = 2)$.
Must have $(p)^2(1-p)^{18}\dfrac{20!}{18!2!}$, with a value of p

A1 awrt 0.0278 or 0.0279.

(b) $1 - P(X \leq 3) = 1 - 0.1071$ M1
$= 0.8929$ A1 2
or

$$1-(0.3)^3(0.7)^{17}\frac{20!}{17!3!}-(0.3)^2(0.7)^{18}\frac{20!}{18!2!}-(0.3)(0.7)^{19}\frac{20!}{19!1!}-(0.7)^{20}$$

M1 Attempting to find 1 – P($X \le 3$)

A1 awrt 0.893

(c) $\frac{10!}{4!6!}(0.8929)^6(0.1071)^4 = 0.0140.$ M1A1ftA1 3

M1 for $k(p)^6(1-p)^4$. They may use any value for p and k can be any number or ${}^{n}C_6p^6(1-p)^{n-6}$

A1ft $\frac{10!}{4!6!}$(their part b)^6(1 – their part b)^4 may write ${}^{10}C_6$ or ${}^{10}C_4$

A1 awrt 0.014

[7]

11. (a) If $X \sim B(n, p)$ and
n is large, $n > 50$ B1
p is small, $p < 0.2$ B1 2
then X can be approximated by Po(np)

(b) P(2 consecutive calls) = 0.01^2 M1
= 0.0001 A1 2

(c) $X \sim B(5, 0.01)$ may be implied B1

P($X > 1$) = 1 – P($X = 1$) – P($X = 0$) M1
= 1 – 5(0.01)(0.99)^4 – (0.99)^5
= 1 – 0.0480298... – 0.95099...
= 0.00098 awrt 0.00098 A1 3

(d) $X \sim B(1000, 0.01)$ may be implied by correct mean and variance B1
Mean = np = 10 B1
Variance = $np(1-p)$ = 9.9 B1 3

(e) $X \sim Po(10)$

P($X > 6$) = 1 – P($X \le 6$) M1
= 1 – 0.1301
= 0.8699 awrt 0.870 A1 2

[12]

12. (a) (i) If $X \sim B(n, p)$ and
n is large or $n > 10$ **or** $np > 5$ **or** $nq > 5$ B1
p is close to 0.5 **or** $nq > 5$ <u>and</u> $np > 5$ B1 2
then X can be approximated by N(np, $np(1-p)$)

	(ii)	mean = np		B1		
		variance = $np(1 - p)$	must be in terms of p	B1	2	
	(b)	$X \sim N(60, 58.2)$ or $X \sim N(60, 7.63^2)$	60, 58.2	B1, B1		
		$P(X \geq 40) = P(X > 39.5)$	using 39.5 or 40.5	M1		
		$= 1 - P\left(z < \pm\left(\frac{39.5 - 60}{\sqrt{58.2}}\right)\right)$	standardising 39.5 or 40 or 40.5 and their μ and σ	M1		
		$= 1 - P(z < -2.68715...)$				
		$= 0.9965$	allow answers in range 0.996 – 0.997	A1dep on both M	5	
	(c)	$E(X) = 60$	may be implied or ft from part (b)	B1ft		
		Expected profit = $(2000 - 60) \times 11 - 2000 \times 0.70$		M1		
		= £19 940.		A1	3	
						[12]
13.	(a)	$P(J \geq 10) = 1 - P(J \leq 9)$	or $= 1 - P(J < 10)$	M1		
		$= 1 - 0.9919$	implies method			
		$= 0.0081$	awrt 0.0081	A1	2	
	(b)	$P(K \leq 1) = P(K = 0) + P(K = 1)$	both, implied below even with '25' missing	M1		
		$= (0.73)^{25} + 25(0.73)^{24}(0.27)$	clear attempt at '25' required	M1		
		$= 0.00392$	awrt 0.0039 implies M	M1	3	
						[5]
14.	(a)	Let W represent the number of white plants.				
		$W \sim B(12,0.45)$	use of	B1		
		$P(W = 5) = P(W \leq 5) - P(W \leq 4)$	$^{12}C_5 0.45^5 0.55^7$ or equivalent award B1M1	M1		
		$= 0.5269 - 0.3044$	values from correct table implies B			
		$= 0.2225$	awrt 0.222(5)	A1	3	
	(b)	$P(W \geq 7) = 1 - P(W \leq 6)$	or $= 1 - P(W < 7)$	M1		
		$= 1 - 0.7393$	implies method			
		$= 0.2607$	awrt 0.261	A1	2	
	(c)	P(3 contain more white than coloured)				
		$= \frac{10!}{3!7!}(0.2607)^3(1 - 0.2607)^7$	use of B, n = 10	M1A1ft		
		$= 0.256654\ldots$	awrt 0.257	A1	3	

	(d)	mean = np = 22.5 ; var = npq = 12.375		B1B1	
		$P(W > 25) \approx \text{P}\left(Z > \frac{25.5 - 22.5}{\sqrt{12.375}}\right)$ ± standardise with σ and μ; ±0.5 c.c.		M1;M1	
		$\approx \text{P}(Z > 0.8528..)$	awrt 0.85	A1	
		$\approx 1 - 0.8023$	'one minus'	M1	
		≈ 0.1977	awrt 0.197 or 0.198	A1	7

[15]

15.	(a)	$\text{P}(L < -2.6) = 1.4 \times \frac{1}{8} = \frac{7}{40}$ or 0.175 or equivalent		B1	1
	(b)	$\text{P}(L < -3.0 \text{ or } L > 3.0) = 2 \times \left(1 \times \frac{1}{8}\right) = \frac{1}{4}$		M1;A1	2
		M1 for 1/8 seen			
	(c)	P (within 3mm) = $1 - \frac{1}{4} = 0.75$ B(20,0.75)	recognises binomial	B1	
		Using B(20,p)		M1	
		Let X represent number of rods within 3mm			
		$\text{P}(X \le 9 / \text{p} = 0.25)$ or $1 - \text{P}(X \le 10 / \text{p} = 0.75)$		M1	
		= 0.9861		A1	4
		awrt 0.9861			

[7]

16.	(a)	Binomial		B1	1
		Let X represent the number of green mugs in a sample			
	(b)	$X \sim \text{B}(10, 0.06)$		B1	
		may be implied or seen in part a			
		$\text{P}(X = 3) = {}^{10}C_3(0.06)^3(0.94)^7$		M1	
		${}^{10}C_3(\text{p})^3(1 - \text{p})^7$			
		0.016808....		A1	3
		awrt 0.0168			

(c) Let X represent number of green mugs in a sample of size 125

(i) $X \sim P_0(125 \times 0.06 = 7.5)$ may be implied B1

$(10 \le X \le 13) = P(X \le 13) - P(X \le 9)$ M1

$0.9784 - 0.7764$

$= 0.2020$ A1 3

awrt 0.202

(ii) $P(10 \le X \le 13) \approx P(9.5 \le Y \le 13.5)$ where $Y \cup N(7.5, 7.05)$

7.05 B1

9.5, 13.5 B1

$$= P\left(\frac{9.5 - 7.5}{\sqrt{7.05}} \le z \le \frac{13.5 - 7.5}{\sqrt{7.05}}\right)$$

±0.5 M1

stand. M1

both values or both correct expressions.

$= P(0.75 \le z \le 2.26)$ A1

awrt 0.75 and 2.26

$= 0.2147$ A1 6

awrt 0.214 or 0.215

[13]

17. (a) Let X be the random variable the number of heads.

$X \sim \text{Bin}(4, 0.5)$

$P(X = 2) = C_2^4\, 0.5^2 0.5^2$ Use of Binomial including ${}^nCr$ M1

$= 0.375$ or equivalent A1 2

2,4,6 acceptable as use of binomial.

(b) $P(X = 4)$ or $P(X = 0)$ B1

$= 2 \times 0.5^4$ $(0.5)^4$ M1

$= 0.125$ or equivalent A1 3

(c) P(HHT) = 0.5^3 no ^nCr M1

= 0.125 or equivalent A1 2

or

P(HHTT) + P (HHTH)

= 2×0.5^4

= 0.125

[7]

18. X = Po (150 × 0.02) = Po (3) B1, B1(dep)

P(X > 7) = 1 – P(X ≤ 7) M1

= 0.0119 awrt 0.0119 A1

Use of normal approximation max awards B0 B0 M1 A0 in the use 1– p(x < 7.5)

[4]

$z = \dfrac{7.5 - 3}{\sqrt{2.94}} = 2.62$

$p(x > 7) = 1 - p(x < 7.5)$

= 1 – 0.9953

= 0.0047

19. (a) X ~ B(n, 0.04) B1

Implied

E(X) = np M1

Use of np = 5

5 = 0.04n

n = 125 A1 3

125

(b) E(X) = 3
np = 3 B1

sd = $\sqrt{npq}$ = $\sqrt{3(1-0.04)}$ M1

Use of npq

= $\sqrt{2.88}$ A1

$\sqrt{3(1-0.04)}$

= 1.70 A1 4

awrt 1.70

[7]

20. (a) X ~ B(10, p) B1, B1 2

Binomial (10, 0.75)

(b) P(X = 6) = 0.9219 – 0.7759 M1

$P(X \le 6) - P(X \le 5)$

= 0.1460 A1 2

(c) H_0: p = 0.75 (or p = 0.25) B1

Correct H_0

H_1: p < 0.75 (or p > 0.25) B1

One tailed H_1

Under H_0, X ~ B(20, 0.75) (or Y ~ B(20,0.25)) B1

Implied

P($X \le 13$) = 1 – 0.7858 = 0.2142 (or P($Y \ge 7$)) M1, A1

$P(X \le 13)$ and 1 –, 0.2142

Insufficient evidence to reject H_0 as 0.2412 > 0.05
Doctor’s belief is not supported by the sample A1 6

Context

(OR CR $P(X \le 12) = 1 - 0.8982 = 0.1018$
(or $P(Y \ge 8)$)
$P(X \le 11) = 1 - 0.9591 = 0.0409$
(or $P(Y \ge 9)$) *either* (M1, A1)
13 outside critical region
(or 7))

(d) $P(X \le c) \le 0.01$ for $p = 0.75$
(or $P(Y \le 20 - c) \le 0.01$ for $p = 0.25$) M1 A1
$P(X \le 9) = 1 - 0.9961 = 0.0039$ (or $P(Y \ge 11)$) B1
0.9961 or 0.9981

$P(X \le 10) = 1 - 0.9861 = 0.0139$ (or $P(Y \ge 10)$) B1 4
C. R. is [0,9], so greatest no. of patients is 9.

[14]

21. (a) $P(R = 5) = P(R \le 5) - P(R \le 4) = 0.7216 - 0.5155$ M1
Can be implied

$= 0.2061$ A1 2
Answer 0.2061

(OR: ${}^{15}C_5\,(0.3)^5\,(0.7)^{10} = 0.206130\ldots$)

(b) $P(S = 5) = 0.2414 - 0.1321 = 0.\underline{1093}$ B1 1
Accept 0.1093 (AWRT) or 0.1094 (AWRT)

(OR: $\dfrac{7.5^5 e^{-7.5}}{5!} = 0.10937459\ldots$)

(c) $P(T = 5) = 0$ B1 1
cao

[4]

22. Let x represent the number of defective articles
$\therefore X \sim B(10, 0.032)$

(a) $P(X = 2) = \overset{45}{{}^{10}\cancel{C}_2}\ (0.032)^2\,(1 - 0.032)^p$ M1 A1
Use of ${}^nC_r p^r q^{n-5}$
All correct

$= \underline{0.0355234\ldots.}$ A1 3
AWRT 0.0355

(b) Large n, small p $\Rightarrow$ Poisson approximation B1
seen or implied

with $\lambda = 100 \times 0.032 = 3.2$

$P(X < 4) = P(X \le 3) = P(0) + P(1) + P(2) + P(3)$ M1
$P(X \le 3)$ stated or implied

$= e^{-3.2}\left\{1 + 3.5 + \dfrac{(3.2)^2}{2} + \dfrac{(3.2)^3}{6}\right\}$ A1

All correct

<u>NB</u> Normal Approx $\Rightarrow {}^{0}/_{4}$

	= <u>0.602519….</u>	A1	4
	AWRT 0.603		
(c)	*np* & *nq* both > 5 $\Rightarrow$ Normal approximation	M1	
	N Approx		
	With $np = 32$ and $npq = 30.976$	A1	
	Both		
	$P(X > 42) \approx P(Y > 42.5)$ where $Y \sim N(32, 30.976)$	M1	
	Standard		
	$= P(Z > \dfrac{42.5 - 32}{\sqrt{30.976}})$	A1	
	their np, $\sqrt{npq}$		
	All correct		
	$= P(Z > 1.8865\ldots)$	A1	
	AWRT 1.89		
	= <u>0.0294</u>	A1	6
	0.0294 – 0.0297		

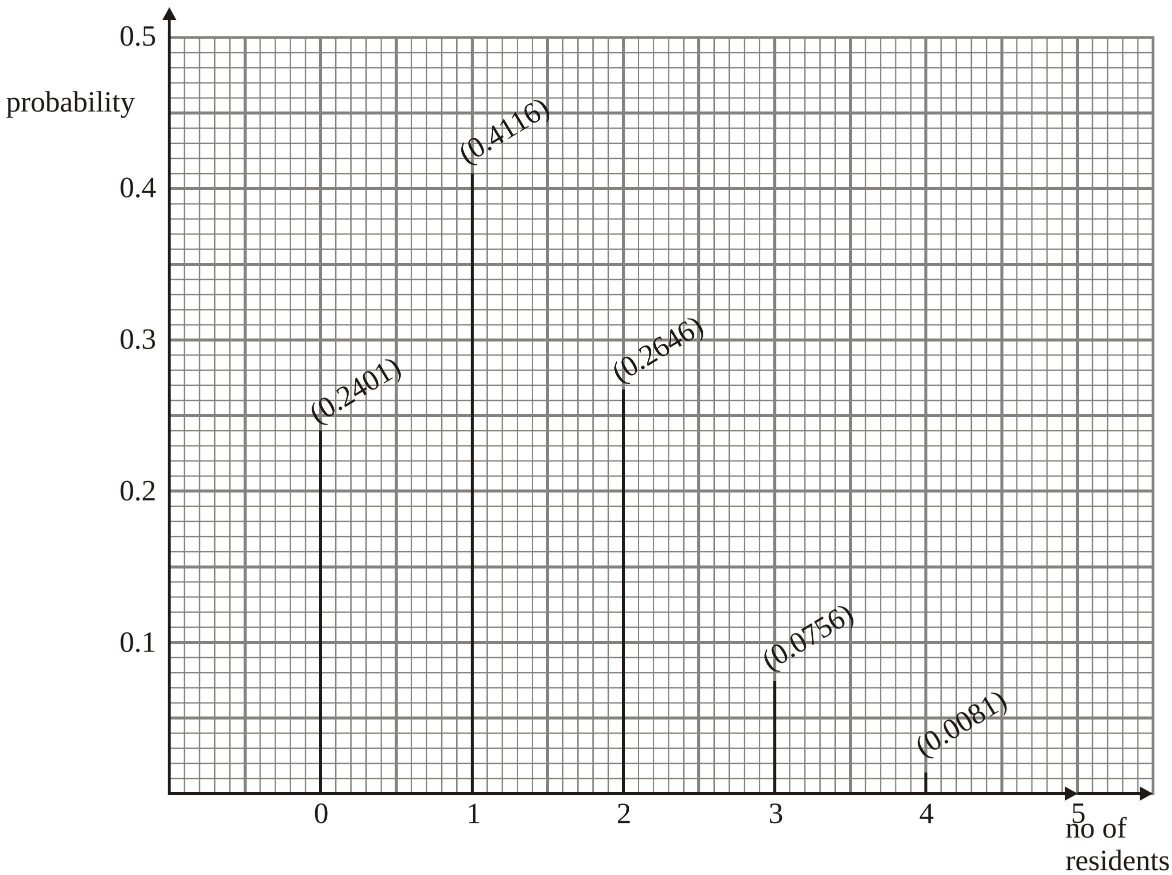
[13]

23.	(a)	Fixed no of trials/ independent trials/ success & failure/ Probab of success is constant any 2	B1B1	2
	(b)	X is rv 'no of defective components $X \sim \text{Bin}(20,0.1)$	B1	1
	(c)	$P(X = 0) = 0.1216$	M1A1	2
		= 0, 0.1216		
	(d)	$P(X > 6) = 1 - P(X \le 6) = 1 - 0.9976 = 0.0024$	M1A1	2
		Strict inequality & 1- with 6s, 0.0024		
	(e)	$E(X) = 20 \times 0.1 = 2$	B1	
		$\text{Var}(X) = 20 \times 0.1 \times 0.9 = 1.8$	B1	2
	(f)	$X \sim \text{Bin}(100,0.1)$	B1	
		Implied by approx used		
		$X \sim P(10)$	B1	
		$P(X > 15) = 1 - P(X \le 15) = 1 - 0.9513 = 0.0487$	M1A1	
		Strict inequality and 1- with 15, 0.0487		
		(OR *$X \sim N(10,9)$, $P(X > 15.5) = 1 - P(Z < 1.83)$ = 0.0336 (0.0334) with 15.5*	*B1M1A1***)**	
		(OR *$X \sim N(10,10)$, $P(X > 15.5) = 1-P(Z<1.74)$*		

		=0.0409 (0.0410) with 15.5	*B1M1A1*)	4
				[13]
24.	(a)	$P(R \geq 4) = 1 - P(R \leq 3) = 0.6533$	M1A1	2
		Require 1 minus and correct inequality		
	(b)	$P(S \leq 1) = P(S = 0) + P(S = 1)$, $e^{-2.71} + 2.71e^{-2.71}$, $= 0.2469$	M1,A1,A1	3
		awrt 0.247		
	(c)	$P(T \leq 18) = P(Z \leq -1.4) = 0.0808$	M1,A1	2
		4 dp, cc no marks		
				[7]
25.	(a)	$p = \frac{1}{2}$	B1	1
	(b)	Binomial distribution is symmetrical	B1	1
	(c)	Since n is large and $p \approx 0.5$ then use normal approximation,	M1	
		Can be implied below		
		$np = 96$ and $npq = 49.92$	A1A1	
		$P(90 \leq X < 105) \approx P(89.5 \leq Y \leq 104.5)$ where $Y \sim N(96{,}49.92)$	M1,	
		±0.5 cc on both		
		$\approx P\left(\frac{89.5 - 96}{\sqrt{49.92}} \leq Z \leq \frac{104.5 - 96}{\sqrt{49.92}}\right)$	M1	
		Standardisation of both		
		$\approx P(-0.92 \leq Z \leq 1.20)$	A1	
		awrt –0.92 & 1.20		
		$\approx 0.7055 - 0.7070$	A1	7
		4dp in range		
				[9]
26.	(a)	n large, p small	B1,B1	2
	(b)	Let X represent the number of people catching the virus, $X \sim B\left(12, \frac{1}{150}\right)$	B1	
		Implied		
		$P(X = 2) = C^{12}_{2}\left(\frac{1}{150}\right)^{2}\left(\frac{149}{150}\right)^{10}$, $= 0.027$	M1A1,A1	4
		Use of Bin including C^{12}_{2}, 0.0027(4) only		

	(c)	$X \sim \text{Po}(np) = \text{Po}(8)$	B1,B1	
		Poisson, 8		
		$\text{P}(X < 7) = \text{P}(X \leq 6) = 0.3134$	M1A1	4
		$X \leq 6$ for method, 0.3134		
				[10]

27.	(a)	$X \sim \text{B}(4, 0.3)$	B1 B1	2
	(b)			



		Use of Binomial	M1	
		All probabilities correct	A1	
		Scales and labels		
		Correct Diagram	A1	3
	(c)	<u>1</u> resident	B1	1
	(d)	$\text{E}(X) = np = \underline{1.2}$	B1	
		$\text{Var}(X) = np(1 - p)$		
		$= 4 \times 0.3 \times 0.7$ (their p)	M1	
		$= \underline{0.84}$	A1	3
				[9]

28. (a) Fixed number; of independent trials — B1; B1
2 outcomes — B1
Probability of success constant — B1 — 4

(b) $P(X = 5) = \frac{2}{7}$; $P(X \neq 5) = \frac{5}{7}$ may be implied — B1; B1 ft
P(5 on sixth throw) = $\left(\frac{5}{7}\right)^5 \times \left(\frac{2}{7}\right)$ $\quad (1-p)^5 p$ — M1 A1 ft
= 0.0531 — A1 — 5

(c) P(exactly 3 fives in first eight throws) = $\binom{8}{3}\left(\frac{2}{7}\right)^3\left(\frac{5}{7}\right)^3$ — M1 A1 ft

use of ${}^nC_r$ needed

= 0.243 — A1 — 3

[12]

29. (a) Let X represent the number of double yolks in a box of eggs — B1
$\therefore X \sim B(12, 0.05)$ — B1
$P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0.8816 - 0.5404 = 0.3412$ — M1 A1 — 3

(b) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.9978 = 0.0022$ — M1 A1 — 2

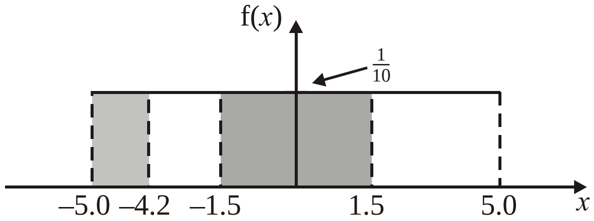
(c) P(only 2) = $C_2^3 (0.3412)^2 (0.6588)^2$ — M1 A1
= 0.230087 — A1 — 3

(d) Let X represent the number of double yolks in 10 dozen eggs
$\therefore X \sim B(120, 0.05) \Rightarrow X = Po(6)$ — B1
$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.8472$ — M1 A1
= 0.1528 — A1 — 4

(e) Let X represent the weight of an egg $\therefore W \sim N(65, 2.4^2)$ — M1
$P(X > 68) = P\left(Z > \frac{68 - 65}{2.4}\right)$ — A1
$= P(Z > 1.25)$ — A1
= 0.1056 — A1 — 3

[15]

30.



(a)	$P(X < -4.2) = \dfrac{0.8}{10} = 0.08$	B1	1
(b)	$P(\lvert X \rvert < 1.5) = \dfrac{3}{10} = 0.3$	M1 A1	2
(c)	Y = no. of lengths with $\lvert X \rvert < 1.5$ $\quad\therefore\ Y \sim B(10, 0.3)$	M1	
	$P(Y > 5) = 1 - P(Y \le 5)$	M1	
	$= 1 - 0.9527 \quad = 0.0473$	A1	3
	R = no. of lengths of piping rejected		
	$R \sim B(60, 0.08) \Rightarrow R \approx \sim Po(4.8)$ 4.8 or 60 × (a)	B1 ft	
	$P(R \le 2) = e^{-4.8}\left[1 + 4.8 + \dfrac{(4.8)^2}{2!}\right]$ Po and ≤ 2, formula	M1, M1 A1 ft	
	(ft for their λ if full expression seen)		
	$= 17.32 \times e^{-4.8} = 0.1425\ldots$		
	(accept awrt 0.143)	A1 cao5	

[11]

31. (a)	X = no. of customers arriving in 10 minute period $X \sim Po(3)$ $P(X \ge 4) = 1 - P(X \le 3) =,\ \ 1 - 0.6472 = 0.3528$	M1 A1	2
(b)	Y = no. of customers in 30 minute period $Y \sim Po(9)$	B1	
	$P(Y \le 7) = 0.3239$	M1 A1	3
(c)	p = probability of no customers in 5 minute period = $e^{-1.5}$	B1	
	C = number of 5 minute periods with no customers		
	$C \sim B(6, p)$	M1	
	$P(C \le 1), = (1-p)^6 + 6(1-p)^5 p$	M1, M1 A1	
	$= 0.59866\ldots$		
	(accept awrt 0.599)	A16	

(d)	W = no. of customers on Wednesday morning		
	$3\frac{1}{2}$ hours = 210 minutes $\quad \therefore\ W \sim \text{Po}(63)$	'63'	B1
	Normal approximation $\quad W \approx \sim \text{N}(63, (\sqrt{63})^2)$		M1 A1
	$\text{P}(W > 49) \approx \text{P}(W \geq 49.5)$	$\pm\frac{1}{2}$	M1
	$= \text{P}\left(Z \geq \dfrac{49.5 - 63}{\sqrt{63}}\right)$		
	standardising		M1
	$= \text{P}(Z \geq -1.7008)$		A1
	$= 0.9554$ (tables)		
	(accept awrt 0.955 or 0.956)		A17

[18]

1. This question was well answered by the majority of candidates with many scoring full marks. There were, of course, candidates who failed to score full marks. This was usually the result of inaccurate details, rather than lack of knowledge. In particular, manipulation of inequalities requires concentration and attention to detail. In part (a) the most common error seen was using $P(X = 3) = P(X \le 4) - P(X \le 3)$.

Parts (b) and (c) were usually correct. The most common error was to find $P(X \le 3)$ rather than $P(X \le 4)$ in part (b). A minority of candidates used the Normal as their approximation in part (d). The simple rule "*n* is large, *p* is small: use Poisson" clearly applies in this case.

2. Part (a) was well answered as no context was required.

In part (b) candidates identified the correct distribution and with much of the working being correct. However although the lower limit for the critical region was identified the upper limit was often incorrect. It is disappointing to note that many candidates are still losing marks when they clearly understand the topic thoroughly and all their work is correct except for the notation in the final answer. It cannot be overstressed that $P(X \le 6)$ is not acceptable notation for a critical region. Others gave the critical region as $6 \le X \le 19$.

In part (c) the majority of candidates knew what to do and just lost the accuracy mark because of errors from part (b) carried forward.

Part (d) tested the understanding of what a critical region actually is, with candidates correctly noting that 8 was outside the critical region but then failing to make the correct deduction from it. Some were clearly conditioned to associate a claim with the alternative hypothesis rather than the null hypothesis. A substantial number of responses where candidates were confident with the language of double-negatives wrote "8 is not in the critical region so there is insufficient evidence to disprove the company's claim". Other candidates did not write this, but clearly understood when they said, more simply "the company is correct".

Part (e) was generally well done with correct deductions being made and the contextual statement being made. A few worked out $P(X = 5)$ rather than $P(X \le 5)$.

3. This question proved to be a very good start to the paper for a large majority of candidates. In general parts (a), (b) and (d) were answered correctly. In part (c) the most common mistake was to use $P(X > 4) = 1 - P(X \le 3)$.

4. Part (a) tested candidates' understanding of the critical region of a test statistic and responses were very varied, with many giving answers in terms of a 'region' or 'area' and making no reference to the null hypothesis or the test being significant. Many candidates lost at least one mark in part (b), either through not showing the working to get the probability for the upper critical value, i.e. $1 - P(X \le 15) = P(X \ge 16) = 0.0064$, or by not showing any results that indicated that they had used B(30, 0.3) and just writing down the critical regions, often incorrectly. A minority of candidates still write their critical regions in terms of probabilities and lose the final two marks. Responses in part (c) were generally good with the majority of candidates making a comment about the observed value and their critical region. A small percentage of responses contained contradictory statements.

5. The majority of candidates achieved full marks on this question with the most common errors caused by difficulties in identifying, interpreting and/or working with the inequalities. In part (b) whilst a few candidates wrote down P(X≤13) they were unable to find this probability correctly. The most common error was to use 1 – P(X≤12)

6. This question was attempted by most candidates with a good degree of success for those who were competent in using the Binomial formula. A number of candidates had difficulty writing 1% as a decimal and used 0.1 in error.

In part (b) the most common errors were to see 'at least 2' translated as P(X >2) or to write P(X ≥ 2) as 1 – P(X ≤ 2). Many final accuracy marks were lost as a result of inadequate rounding in both parts (a) and (b).

In part (c) Po(2.5) ensured many scored at least 2 marks but P(X≤4) – P(X≤1) was a common error.

A normal approximation was seen but not quite as often as in previous years.

7. Candidates did this well on the whole, with the main error being the absence of a continuity correction (which was penalised appropriately.) The question worked well although the 42% mentioned in the question caused some confusion as many took this to be the percentage of students that were female rather than 58%. Very few candidates were unable to make a reasonable attempt at a Normal approximation.

8. This question was generally answered well. A few candidates put the Poisson for (a) and then used Variance = Mean to got 5.5 for the variance. Some candidates rounded incorrectly giving an answer of 5.49 for the variance.

Part (c) was generally answered correctly although a minority of candidates used the normal approximation – most used 2.5 in their standardisation and so got 1 mark out of the 4.

9. Most candidates recognised that this was a Binomial for part (a) but quite a few did not define it completely by putting in the 15 and 0.5. Parts (b) and (c) were answered well .Most candidates are getting much better at the lay out of their solutions and it was good to see that candidates are identifying one-tail tests, although some candidates did not get the hypotheses completely correct; omitting p altogether or using λ was not uncommon. The working for finding 0.0037 or a critical region ≥ 13 was often done well. Those who used a critical region method required more working and seemed to make more errors. It is recommended that this method is not used in Hypothesis testing at this level. There was the usual confusion in interpreting the test. In particular candidates often did not interpret correctly in context. 'The coin is biased' was a common inadequate answer.

10. This question was well answered by the majority of candidates.

(a) This proved relatively straightforward for most, with the occasional response finding $P(X \le 2)$ or using the binomial formula with $x = 3$.

(b) The most common error was interpreting 'more than 3' as $1 - P(X \le 2)$.

(c) Answers to this part reflected good preparation on this topic with a high proportion of successful responses. Candidates who lost marks on this question did so because their response to part (b) was incorrect or they used $n = 20$. A few candidates gave the answer as $P(X = 6) = (\text{answer to part (b)})^6$.

11. This question was accessible to most candidates. Parts (a), (d) and (e) were generally well answered with a small proportion of candidates using Po(10) in part (d) and thus quoting that the mean and variance were the same. Part (b) caused a few problems as some used Po(0.1) and found $P(X = 2)$ or used $X \sim B(2, 0.1)$. In part (c) some candidates still did not correctly use $P(X > 1) = 1 - P(X < 1)$. A similar mistake occurred in part (e) where they used $P(X > 6) = 1 - P(X \le 6)$.
Candidates need to be reminded of the rubric on the front of the question paper. It does say 'appropriate degrees of accuracy'. Many rounded too early and did not realise that an answer to 1sf is not accurate enough. Answers of 0.001 were common in part (c)

12. In part (a) a very large number of candidates thought that np and npq must exceed 5 rather than np and nq or that 'n is large and p is small' A minority of candidates gave the variance in terms of p and q not just p as stated in the question. In part b most candidates were able to apply a continuity correction successfully and use the normal distribution, although there were a sizable few candidates who had difficulty knowing whether to do '1 – or not'. It is recommended that candidates draw a diagram to help them decide which is the area required. It was rather disturbing that many candidates could not apply simple logical arithmetic to part (c). The most common error was to fail to take into account that the faulty DVDs still cost something to make even if they are not sold.

13. This was usually completely correct with very few errors.

14. Many candidates achieved full marks for this question, demonstrating a good understanding of, and ability to use the binomial distribution and its approximation by the normal distribution. Part (a) was usually answered well with candidates either using the formula or tables. Weaker candidates still failed to appreciate that $P(X = 5) = P(X \le 5) - P(X \le 4)$. A common problem for weaker candidates in part (b) was to translate the concept of 'more white than coloured' into a correct probability statement. Of those that correctly stated that $P(X \ge 7)$ was required, a few were unable to equate this with $1 - P(X \le 6)$. In part (c) a common error was to use the original p=0.45 rather than the carried forward solution to part (b). Most of those identifying the correct distribution had little problem in calculating the probability accurately. The increasing number of candidates that are able to make a sensible attempt at a normal approximation to a binomial distribution suggests that there is an encouraging awareness of the importance of approximations from simple distributions. Most candidates calculated the parameters correctly here and were able to standardise using a continuity correction, although there is still an appreciable number who omit to use the 0.5 correction. A common error was to forget the $1 - \Phi(z)$ and stated $\Phi(z)$ as their solution.

15. Part (a) was mostly correct although there were some very long-winded solutions seen. Drawing a diagram (as is often the case) was a successful approach to use. Part (b) was generally answered correctly although if integration was used the solution tended to be lengthy. Common wrong answers were 1/8 and ¾ were common wrong answers. Weaker candidates clearly did not understand the use of the word “or” in probability and failed to add the probabilities for the two parts. In part (c) whilst most candidates recognised a binomial situation and found the correct value for p, few candidates were able to cope with a value of $p > 0.5$. It was common to see $P(X > 10)$ given X~B(20, 0.75) interpreted as $P(Y \leq 10)$, or $1 - P(Y \leq 10)$, given X~B(20, 0.25). There was poor understanding of how to use the binomial tables for situations in which p is greater than 0.5.

16. This was well answered by almost all candidates and many correct solutions were seen. A few candidates tried to use Poisson rather than Binomial for parts (a) and (b). In part (b) a few candidates used B(10, 0.6) instead of B(10, 0.06). In part (c)(i) most errors occurred because candidates did not understand what was meant by “between 10 and 13 inclusive” The most common wrong answer was in using $P(10 \leq X \leq 13) = P(X \leq 13) - P(X \leq 10)$ instead of $P(X \leq 13) - P(X \leq 9)$ Another fairly common error was using $P(X \leq 13) - (1 - P(X \leq 10))$ Some candidates tried to use a continuity correction in this Poisson approximation. Part (c)(ii) was often correct the most common errors being to use 7.5 instead of 7.05 for the variance and to use an incorrect continuity correction.

17. A few candidates did not recognise tossing a coin as being a Binomial situation and thus gained no marks. Most candidates coped well with the question and full marks were not uncommon. The main error came in part (b) where many candidates forgot to consider both possibilities and got an answer of 0.0625. In part c the most common error was to try to use a binomial approach.

18. Most candidates correctly used a Po(3) distribution although a significant minority attempted to used a Normal distribution**.** The most common error was using $P(X > 7) = 1 - P(X \leq 6)$

19. Most candidates attempted this question and achieved good marks. Part (a) was completed well. There were a large number of responses which relied on an informal, intuitive approach involving ratios. Part (b) was less well done and provided a variety of responses. A substantial minority of candidates had failed to read the question properly. They assumed that the “second random sample” had the same size as the first random sample. These candidates also assumed that the proportion of people with green eyes had somehow changed. As a consequence common errors included using $n = 125$ or using $p = 3/125$. When $np = 3$ was used correctly then a common error was to calculate $npq = 2.88$ and leave this as a final answer.

20. This proved to be a difficult question but a lot of good mathematics was seen. In part (a) most candidates obtained both marks. Most candidates obtained the first mark in part (b), but the second proved elusive for some. A final answer of 0.146 was common. Part (b) was important for focusing the mind in part (c) on a Binomial distribution with $p>0.5$, i.e. not in the tables. However, there was still some confusion evident. A majority of candidates earned the first three marks. Most candidates who proceeded beyond this point decided to adopt the Bin (20, 0.25) approach. There were problems identifying the correct inequality with a common misconception was that the correct value was $P(X = 13)$ or $P(X = 7)$. Other less common errors were $P(X < 13)$ and $P(X \geq 13)$ or their counterparts. There were a substantial number of responses where candidates decided to use a Normal approximation and some candidates even used a Poisson distribution (Po(7.5)). Part (d) was fairly demanding, but there were candidates who provided answers that were not only correct but also clear and concise. The remainder struggled. Many candidates failed to distinguish between $X \sim B(20,0.75)$ and $Y \sim B(20,0.25)$, using X to denote both distributions. This caused confusion in the interpretation of their solutions, when candidates obtained a 'final' value of eleven. This was a Y value and needed conversion to an X value.

21. Candidates knew how to answer parts (a) and (b) but many did not work to sufficient accuracy. If they used their calculator instead of the tables they were expected to give their answer to the same accuracy as the tables. Too many of them did not read part(c) carefully enough. The random variable T was defined to be normally distributed and thus $\text{P}(T = 5) = 0$.

22. This question was a good source of marks for many of the candidates, with many of them gaining full marks. For those that did not gain full marks, the common errors were premature approximation; wrong interpretation of 'fewer than 4';ignoring the continuity correction and in part (c) using a Poisson approximation and then a normal approximation to this Poisson approximation.

23. This was a good source of marks for a large majority of candidates. Many were able to write down two conditions for a Binomial distribution, with some writing down all four conditions for good measure. However some candidates muddled up the concepts of *trial*, *event* and *outcome*. The response of 'a fixed number of events' was given no credit. Parts (b), (c), (d) and (e) were usually well answered. In part (f), successful candidates either applied a Po (10) or a N(10, 9) approximation. Some of the candidates who used the N(10, 9) approximation did not apply the correct continuity correction.

24. The whole question was generally very well answered, by far the most common error was the use of a continuity correction in part (c). A small number of candidates didn't realise that part (b) required the sum of two probabilities.

25. In part (a), a minority of candidates offered the correct response of $p = \frac{1}{2}$. Only a few of these candidates, however, were able to give the correct reason. In part (c) the majority of candidates produced solutions giving the correct answer. The most common error was the incorrect application of the continuity correction with candidates generally losing the final two accuracy marks.

26. This was a well answered question with many candidates scoring full marks. In part (a), many candidates realised the conditions of a *large value of n* and a *small value of p* when approximating the Binomial Distribution by the Poisson distribution. One common error in part (b) was for candidates to apply the Poisson approximation when the number of trials was only twelve, even though these candidates were able to write down the appropriate conditions in part (a).

27. In part (a) many candidates were able to recognize the binomial distribution and give its parameters. In part (b) a sizeable number of candidates evaluated the binomial probabilities but few drew a correct diagram. Common errors included the omission of labels from the graph and the joining up of each of the probabilities to form a polygon. In part (c) most students wrote down the mode as 1, so gaining the mark, but almost all arrived at it by calculating a mean of 1.2 and then saying it needed to be a whole number.

28. In part (a) many candidates were able to state the four conditions for a binomial distribution but some lost marks for stating that the events were independent rather than the trials were independent. In part (b) many candidates were able to calculate the probability of gaining a 5 correctly, but a common error was to use the binomial distribution to calculate the probability of gaining a five on the sixth throw. Many candidates recovered and correctly calculated the probability of gaining 3 fives in 8 throws.

29. This was the question which most candidates found to their liking and which gave them ample opportunity to score high marks. Most were successful in parts (a) and (b), although some chose to exercise their calculator in part (b) rather than use tables. Part (c) was usually done well, but here again a mark was easily lost by not following the rubric. Part (d) held no terrors for most, but in part (e) too many candidates, including some of the better ones, fell into the continuity correction trap.

30. No Report available for this question.

31. No Report available for this question.